

Math 131B-2: Homework 9

Due: June 6, 2014

1. Read Tao Sections 16.3-5.
2. Prove Pythagoras' Identity: If $\langle f, g \rangle = 0$, then $\|f + g\|_2^2 = \|f\|_2^2 + \|g\|_2^2$.
3. Prove that the convolution $f * g$ of two continuous \mathbb{Z} -periodic function is continuous. Hint: You will need to use that f is bounded and g is uniformly continuous.
4. Do Tao problem 16.2.3. Hint: You can't do this problem with a single function g ; if you try to, you will sometimes get negative values of c and d when you solve. Instead, you need to be able to produce functions g with $\sup_{[0,1]} g = k$ and $\int_0^1 g = \ell$ such that $kA^2 - \ell B^2 > 0$.
5. Do Tao problems 16.2.6, 16.5.1, 16.5.2, and 16.5.4. Note the existence of a typo in 16.5.4: It should say $\widehat{f}'(n) = 2\pi i n \widehat{f}(n)$.

Caveat: This assignment is not as short as it looks. Several of the problems above have multiple parts.